KANTONALNO TAKMIČENJE IZ MATEMATIKE
I RAZRED
Srednjobosanski kanton, školska 2014/15. godina

ZADACI

1.Ako su a , b i c dužine stranica trougla, dokazati da je $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}<2$

2.Ugao u vrhu B trougla ABC je dva puta veći od ugla u vrhu A, a težišna duž CD je normalna na simetralu ugla u vrhu B.Izračunati uglove trougla ABC.

3.Neka je $A(n)=(n^2+5n)(n^2+5n+10)$. Dokazati da je za svaki prirodan broj n vrijednost izraza A(n) broj koji je djeljiv sa 24.

4.Dat je skup različitih tačaka $\{A_1, A_2, A_3, \ldots\ldots A_{2014}, A_{2015}\}$ koje pripadaju kružnici k.Koliko pravih u toj ravni određuje zadani skup tačaka?

5.Odrediti sve cijele brojeve p za koje je i broj $M=\frac{p^3-p^2+3}{p-1}$ takođe cio broj.

Vrijeme predviđeno za izradu zadataka je 120 minuta.
Dozvoljena je upotreba samo pribora za crtanje i pisanje.

RJEŠENJA I RAZRED

1. a, b, c su dužine stranica trougla to je

$b+c>a \;|+(b+c)$

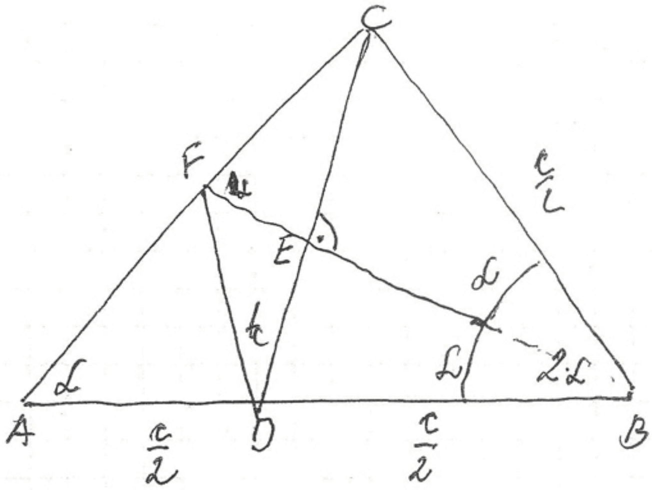
$2(b+c)>a+b+c \Rightarrow \frac{1}{b+c}<\frac{2}{a+b+c} \;|\cdot a \Rightarrow \frac{a}{b+c}<\frac{2a}{a+b+c}$

na isti način $\frac{1}{a+c}<\frac{2}{a+b+c} \;|\cdot b \Rightarrow \frac{b}{a+c}<\frac{2b}{a+b+c}$

$\frac{1}{b+c}<\frac{2}{a+b+c} \;|\cdot c \Rightarrow \frac{c}{b+c}<\frac{2c}{a+b+c}$ $\quad +$

$$\Rightarrow \frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{b+c}<\frac{2a+2b+2c}{a+b+c}=\frac{2(a+b+c)}{a+b+c}=2$$

2.



$\Delta DBE \cong BEC$ (USU)

$\Rightarrow \sphericalangle CDB = \sphericalangle DCB$

$BD = BC$

$\Delta FDE \cong \Delta ECF$ (SUS) $\Rightarrow$

$FD = FC$

pa je $\Delta DBF \cong \Delta FBC$ (SSS

$\Rightarrow \sphericalangle FDB = \sphericalangle FCB$

$\sphericalangle FAD = \sphericalangle FBA = \alpha \Rightarrow$

ΔABF jednakokraki

$AD = DB = \frac{c}{2}$ pa je

FD visina ΔABF a

ugao $\sphericalangle FDB = 90^\circ \Rightarrow$

$\Rightarrow \sphericalangle ACB = 90^\circ$ pa je

$\sphericalangle + \sphericalangle B = 90^\circ$

$\alpha + 2\alpha = 90^\circ$

$\alpha = 30^\circ$ $\quad \beta = 60^\circ$

3) $A(n) = (n^2+5n)(n^2+5n+10) \quad 24 \mid A(n) \quad n \in N$

$$= (n^2+5n)^2 + 10(n^2+5n) + 25 - 1 - 24 =$$
$$= (n^2+5n+5)^2 - 1 - 24 =$$
$$= (n^2+5n+4)(n^2+5n+6) - 24 =$$
$$= (n^2+n+4n+4)(n^2+2n+3n+6) - 24 =$$
$$= (\underbrace{(n+1)(n+4)(n+2)(n+3)}) - 24$$

četiri uzastopna broja
pa je jedan djeljiv sa 2, jedan
sa 3, a jedan sa 4, odnosno
$2 \cdot 3 \cdot 4 = \underline{24}$ - razlika djeljiva sa
24 pa je i $24 \mid A(n)$

4. $\{A_1, A_2, A_3, \dots A_{2014}, A_{2015}\} \in k$
predstavljaju vrhove poligona ~~sa~~ pa
je broj vrhova 2015., ako sa x označimo
broj traženih pravi onda je
X = broj stranica + broj dijagonala

$$x = n + D(n)$$
$$X = 2015 + \frac{n(n-3)}{2} = 2015 + \frac{2015 \cdot \cancel{2012}^{1006}}{\cancel{2}_1}$$
$$= 2015 + 212590 =$$
$$= \underline{214605}$$

$$\begin{array}{r} 2015 \cdot 1006 \\ \hline 12090 \\ 212590 \\ + \quad 2015 \\ \hline 214605 \end{array}$$

5. $p \in Z \quad M \in Z$

$$M = \frac{p^3 - p^2 + 3}{p-3} =$$
$$= p^2 + 2p + 6 + \frac{21}{p-3} \Rightarrow$$

$$\begin{array}{l} (p^3 - p^2 + 3) : (p-3) = p^2 + 2p + 6 \\ \underline{p^3 - 3p^2} \\ \quad 2p^2 + 3 \\ \quad \underline{2p^2 - 6p} \\ \qquad 6p + 3 \\ \qquad \underline{6p - 18} \\ \qquad\quad 21 \end{array}$$

$21 \mid p-3 \Rightarrow p-3 = \{1, 3, 7, 21, -1, -3, -7, -21\}$

$1^\circ \; p-3 = 1 \Rightarrow p = 4 \quad M = 16+8+6+21 = 51$
$2^\circ \; p-3 = -1 \Rightarrow p = 2 \quad M = 4+4+6-21 = -7$
$3^\circ \; p-3 = 3 \Rightarrow p = 6 \quad M = 36+12+6+7 = 61$
$4^\circ \; p-3 = -3 \Rightarrow p = 0 \quad M = 6-7 = -1$

$5^\circ \quad p-3=7 \Rightarrow p=10 \Rightarrow n=100+20+6+3=129$

$6^\circ \quad p-3=-7 \Rightarrow p=-4 \Rightarrow n=16-8+6-3=11$

$7^\circ \quad p-3=21 \Rightarrow p=24 \Rightarrow n=576+48+6+1=631$

$8^\circ \quad p-3=-21 \Rightarrow p=-18 \Rightarrow n=324-36+6-1=293$

KANTONALNO TAKMIČENJE IZ MATEMATIKE
II RAZRED
Srednjobosanski kanton, školska 2014/15. godina

ZADACI

1.U skupu cijelih brojeva riješiti jednačinu : $x^2 - y^2 = 2y + 13$

2.Pojednostaviti izraz za a > 0 , b >0 ,a ≠ b :

$$\frac{\left(\frac{a-b}{\sqrt{a}+\sqrt{b}}\right)^3+2a\sqrt{a}+b\sqrt{b}}{3a^2+3b\sqrt{ab}}+\frac{\sqrt{ab}-a}{a\sqrt{a}-b\sqrt{a}}.$$

3.Za koje realne vrijednosti parametra m rješenja jednačine $(1-3m)x^2+2x+2-8m=0$ su:

a) realna i različita
b) realna i jednaka
c) konjugovano kompleksna ?

4.Ako u trouglu ABC važi $\alpha=2\beta$, onda je $a^2 - b^2 = bc$. Dokazati.

5.Izračunati : $\sqrt{444\ldots444+11\ldots11-66\ldots6}$;ako je 2n četvorki,n+1 jedinica i n šestica u zapisu brojeva.

Vrijeme predviđeno za izradu zadataka je 120 minuta.
Dozvoljena je upotreba samo pribora za crtanje i pisanje.

II RAZRED.

1. $x, y \in \mathbb{Z}$

$x^2 - y^2 = 2y + 13$

$x^2 - y^2 - 2y - 1 = 12$

$x^2 - (y+1)^2 = 12$

$(x - y - 1)(x + y + 1) = 12$ – oba faktora su iste parnosti pa ćemo uzimati u obzir slučajeve
$12 = 2 \cdot 6 = -2 \cdot (-6)$

1° $x - y - 1 = 2$
$x + y + 1 = 6 \Rightarrow 4 + y + 1 = 6$
$2x = 8$
$\boxed{x = 4}$ $\boxed{y = 1}$

2° $x - y - 1 = 6$
$x + y + 1 = 2 \Rightarrow y = 2 - 5$
$2x = 8$
$\boxed{x = 4}$ $\boxed{y = -3}$

3° $x - y - 1 = -2$
$x + y + 1 = -6 \Rightarrow y = -6 + 3$
$2x = -8$
$\boxed{x = -4}$ $\boxed{y = -3}$

4° $x - y - 1 = -6$
$x + y + 1 = -2 \Rightarrow y = -2 + 3$
$2x = -8$
$\boxed{x = -4}$ $\boxed{y = 1}$

2. $$\frac{\left(\frac{a-b}{\sqrt a+\sqrt b}\right)^3 + 2a\sqrt a + b\sqrt b}{3a^2+3b\sqrt{ab}} + \frac{\sqrt{ab}-a}{a\sqrt a - b\sqrt a} =$$

$$= \frac{\left(\frac{(\sqrt a-\sqrt b)(\sqrt a+\sqrt b)}{\sqrt a+\sqrt b}\right)^3 + 2a\sqrt a + b\sqrt b}{3\sqrt{a^4}+3\sqrt{ab^3}} + \frac{\sqrt{ab}-\sqrt{a^2}}{\sqrt a(a-b)} =$$

$$= \frac{(\sqrt a-\sqrt b)^3+2a\sqrt a+b\sqrt b}{3\sqrt a(\sqrt{a^3}+\sqrt{b^3})} + \frac{\cancel{\sqrt a}(\sqrt b-\sqrt a)}{\cancel{\sqrt a}(\sqrt a-\sqrt b)(\sqrt a+\sqrt b)} =$$

$$= \frac{\sqrt a^3 - 3\sqrt a^2\cdot\sqrt b + 3\sqrt a\cdot\sqrt b^2 - \sqrt b^3 + 2a\sqrt a + b\sqrt b}{3\sqrt a(\sqrt a+\sqrt b)(a-\sqrt{ab}+b)} + \frac{-1}{\sqrt a+\sqrt b} =$$

$$= \frac{a\sqrt a - 3a\sqrt b + 3b\sqrt a - \cancel{b\sqrt b} + 2a\sqrt a + \cancel{b\sqrt b}}{3\sqrt a(\sqrt a+\sqrt b)(a-\sqrt{ab}+b)} - \frac{1}{\sqrt a+\sqrt b} =$$

$$= \frac{3a\sqrt a - 3a\sqrt b + 3b\sqrt a}{3\sqrt a(\sqrt a+\sqrt b)(a-\sqrt{ab}+b)} - \frac{1}{\sqrt a+\sqrt b} =$$

$$= \frac{\cancel{3\sqrt a(a-\sqrt{ab}+b)}}{\cancel{3\sqrt a}(\sqrt a+\sqrt b)\cancel{(a-\sqrt{ab}+b)}} - \frac{1}{\sqrt a+\sqrt b} =$$

$$= \frac{1}{\sqrt a+\sqrt b} - \frac{1}{\sqrt a+\sqrt b} = 0$$

3. $(1-3m)x^2+2x+2-8m=0$

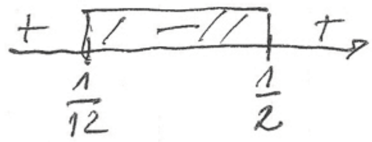
$a=1-3m$
$b=2$
$c=2-8m$

$D=b^2-4ac$
$D=4-4(1-3m)(2-8m)$
$D=4-4(2-8m-6m+24m^2)$
$D=4-4(2-14m+24m^2)$

a) $D>0$

$4-4(2-14m+24m^2>0\ |:4$
$1-2+14m-24m^2>0\ |\cdot(-1)$
$24m^2-14m+1<0$
$D=196-96$
$D=100$
$\sqrt{D}=10$

$m_{1/2}=\frac{14\pm10}{48}$
$m_1=\frac{4}{48}=\frac{1}{12}$
$m_2=\frac{24}{48}=\frac{1}{2}$



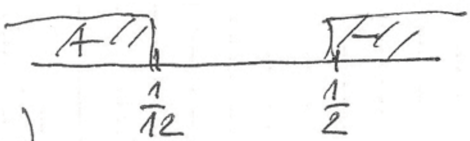
$m\in\left(\frac{1}{12},\frac{1}{2}\right)$

2° $D=0$

$24m^2-14m+1=0$
$m_1=\frac{1}{12}\quad m_2=\frac{1}{2}$

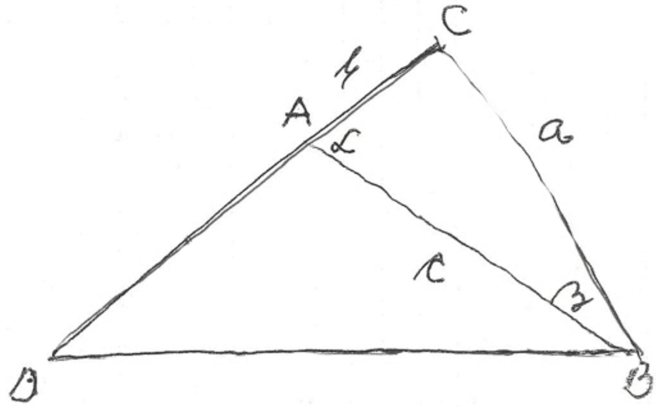
3° $D<0$

$24m^2-14m+1>0$



$m\in\left(-\infty,\frac{1}{12}\right)\cup\left(\frac{1}{2},+\infty\right)$

4. $\alpha=2\beta \Rightarrow a^2-b^2=bc$



Stranicu CA produžimo preko A do tačke D tako da je $AD=AB=c$ tada je $\measuredangle ADB=\measuredangle ABD=\frac{1}{2}\alpha=\beta$ pa su $\triangle ABC\sim\triangle BDC$ i važi

$BC:AC=CD:BC$

ili $a:b=(b+c):a \Rightarrow$

$a^2=b(b+c)$
$a^2=b^2+bc$
$\boxed{a^2-b^2=bc}$

5. $$\sqrt{\underbrace{444\ldots444}_{2n}+\underbrace{11\ldots11}_{n+1}-\underbrace{66\ldots6}_{n}}=$$

$$1=\frac{1}{9}\cdot(10^1-1)$$
$$11=\frac{1}{9}\cdot(10^2-1)$$
$$\underbrace{11\ldots11}_{n\ jd}=\frac{1}{9}(10^n-1)$$

$$=\sqrt{\frac{4}{9}(10^{2n}-1)+\frac{1}{9}(10^{n+1}-1)-\frac{6}{9}\cdot(10^n-1)}=$$

$$=\sqrt{\frac{4\cdot10^{2n}-4+10\cdot10^n-1-6\cdot10^n+6}{9}}=$$

$$=\sqrt{\frac{4\cdot10^{2n}+4\cdot10^n+1}{9}}=\frac{\sqrt{(2\cdot10^n)^2+2\cdot10^n\cdot2+1}}{3}=$$

$$=\frac{\sqrt{(2\cdot10^n+1)^2}}{3}=\frac{2\cdot10^n+1}{3}=\underbrace{66\ldots\ 66}_{n-\check{s}estica}7$$

KANTONALNO TAKMIČENJE IZ MATEMATIKE
III RAZRED
Srednjobosanski kanton, školska 2014/15. godina

ZADACI

1.Izračunati poluprečnik upisanog kruga trougla u kome je

$a + b = 8$, $h_c = \frac{15\sqrt{3}}{14}$, $\gamma = 120^0$.

2.Riješiti jednačinu :

$$sin^4x + \ sin^2xcos^2x + \ cos^4 = cos4x$$

3.Stranice trougla leže na pravima čije su jednačine u koordinatnom sistemu :
$Y+8=0$, $3x-4y+7=0$, $4x+3y-24=0$.Naći jednačinu upisane kružnice tog trougla.

4.Za koju vrijednost nepoznate je slijedeći izraz realan:

$A=\sqrt{log\frac{2x+1}{2x-1}}$?

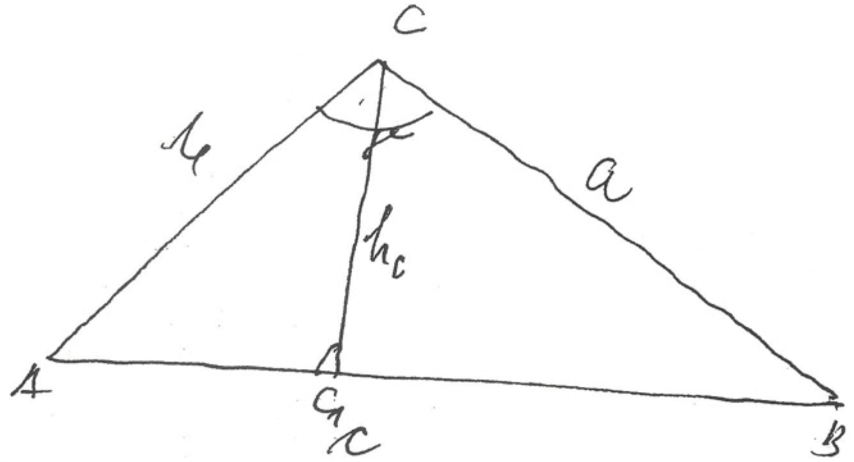
Vrijeme predviđeno za izradu zadataka je 120 minuta.
Dozvoljena je upotreba samo pribora za crtanje i pisanje.

III RAZRED

1. $a+b=8$

$h_c = \frac{15\sqrt{3}}{14}$

$\gamma = 120^\circ$



$ab \sin\gamma = c \cdot h_c$

$ab \cdot \frac{\sqrt{3}}{2} = c \cdot \frac{15\sqrt{3}}{14}$

$ab = c \cdot \frac{15}{7}$

$ab = \frac{15}{7} c$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$c^2 = a^2 + b^2 + 2ab \cdot \frac{1}{2}$

$c^2 = a^2 + b^2 + ab$

$c^2 = a^2 + b^2 + 2ab - ab$

$c^2 = (a+b)^2 - ab$

$c^2 = 64 - \frac{15}{7} c \,/\cdot 7$

$7c^2 + 15c - 448 = 0$

$D = 225 + 12544$

$\sqrt{D} = 113$

$c_{1/2} = \frac{-15 \pm 113}{14}$

$\boxed{c = 7}$

$a+b=8$

$P = r \cdot s$

$r = \frac{P}{s}$

$r = \frac{\frac{15\sqrt{3}}{\cancel{4}^{2}}}{\frac{15}{2}}$

$r = \frac{\sqrt{3}}{2}$

$P = \frac{1}{2} c \cdot h_c$

$P = \frac{1}{2} \cancel{7}^{1} \cdot \frac{15\sqrt{3}}{\cancel{14}_{2}}$

$P = \frac{15\sqrt{3}}{4}$

$s = \frac{a+b+c}{2}$

$s = \frac{8+7}{2}$

$s = \frac{15}{2}$

2. $\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x = \cos 4x$

$$(\sin^2 x + \cos^2 x)^2 - \sin^2 x \cos^2 x = \cos 4x$$

$$1 - \sin^2 x \cos^2 x = \cos 4x \quad *$$

$$\sin^2 x \cos^2 x = \frac{\sin^2 2x}{4} = \boxed{\frac{1-\cos 4x}{8}}$$

$$\sin^2 2x = 1 - \cos^2 2x \quad \Big| + \sin^2 2x$$

$$2\sin^2 2x = 1 - \cos^2 2x + \sin^2 2x$$

$$\sin^2 2x = \frac{1 - (\cos^2 2x - \sin^2 2x)}{2}$$

$$\sin^2 2x = \frac{1 - \cos 4x}{2}$$

$$* \quad 1 - \frac{1-\cos 4x}{8} = \cos 4x$$

$$\frac{8 - 1 + \cos 4x}{8} = \cos 4x \quad \Big| \cdot 8$$

$$7 + \cos 4x = 8\cos 4x$$

$$\cos 4x = 1$$

$$4x = \mathrm{ac}\cos(1)$$

$$4x = 0 + 2k\pi \quad \Big| : 4$$

$$\boxed{x = \frac{k\pi}{2}}$$

3. $y + 8 = 0$

$3x - 4y + 7 = 0$

$4x + 3y - 24 = 0$

$k_u = ?$

$y_1 = -8$

$4y = 3x + 7$

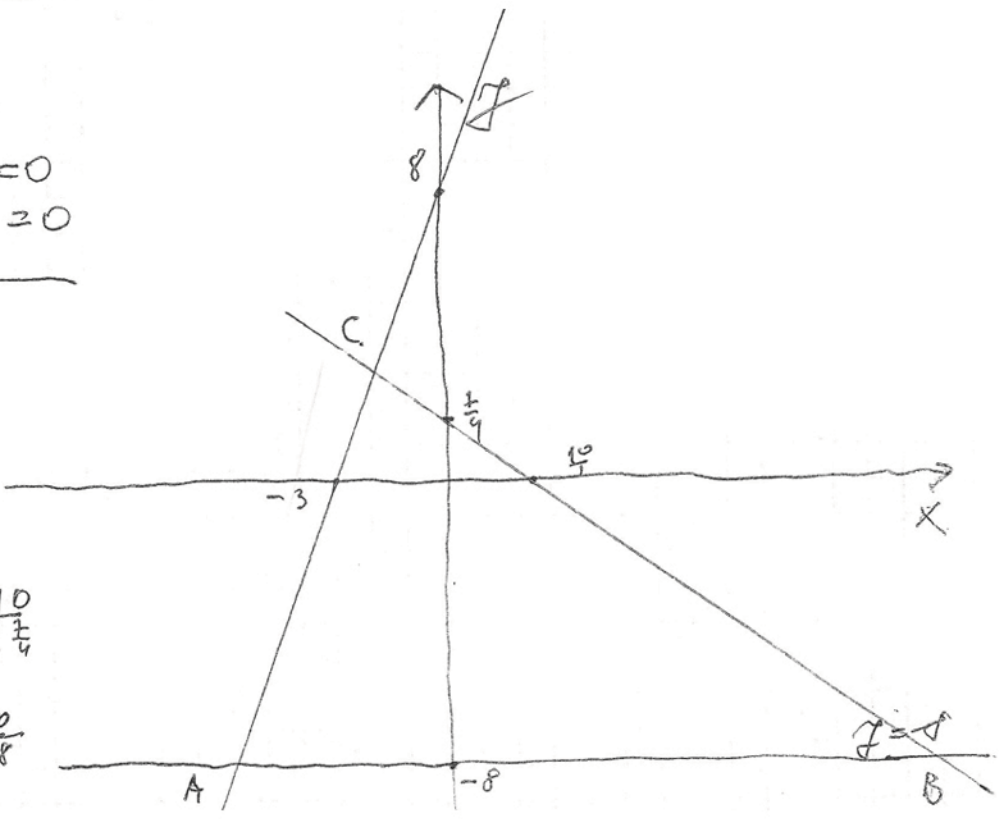
$y_2 = \frac{3}{4}x + \frac{7}{4}$

x	1	0
y	$\frac{10}{4}$	$\frac{7}{4}$

$3y = -4x + 24$

$y_3 = -\frac{4}{3}x + 8$

x	−3	0
y	4	8



$y+8=0$

C_1 $3x-4y+7=0\ /\cdot 3$ $\Rightarrow 4y=3x+7$
$4x+3y-24=0\ /\cdot 4$

$4y=9+7$

$4y=16$

$y=4$

$9x-12y+21=0$
$16x+12y-96=0$

$25x-75=0\ /:25$

$x-3=0$

$x=3$

$C\ (3,4)$

A. y_1 i y_3

$y=-8 \Rightarrow 4x-24-24=0$

$4x=48$

$x=12$

$B\ (12,-8)$

$y=-8$

$3x+\cancel{32}+7=0$

$3x+39=0$

$3x=-39$

$x=-13$

$A\ (-13,-8)$

$\overline{AB}=25$

$\overline{BC}=\sqrt{81+144}$

$\overline{BC}=\underline{15}$

$\overline{AC}=\sqrt{256+144}$

$\boxed{\overline{AC}=20}$

$k_1\cdot k_2=-1$

$\frac{3}{4}\cdot\frac{-4}{3}=-1$

$-1=-1 \Rightarrow \measuredangle=90^\circ \Rightarrow r=\frac{P}{s}=\frac{\frac{1}{2}\cdot 20\cdot 15}{\frac{1}{2}\cdot(25+20+15)}$

$r=\frac{20\cdot 15}{60}$

$\boxed{r=5}$

4. $\sqrt{\log \frac{2x+1}{2x-1}} \Rightarrow$

$\log \frac{2x+1}{2x-1} \geq 0 \quad \wedge \quad \frac{2x+1}{2x-1} > 0$

$\frac{2x+1}{2x-1} \geq 1$

$\frac{2x+1}{2x-1} - 1 \geq 0$

$\frac{2x+1-2x+1}{2x-1} \geq 0$

$\frac{2}{2x-1} \geq 0 \Rightarrow$

$2x-1 \geq 0$

$2x \geq 1$

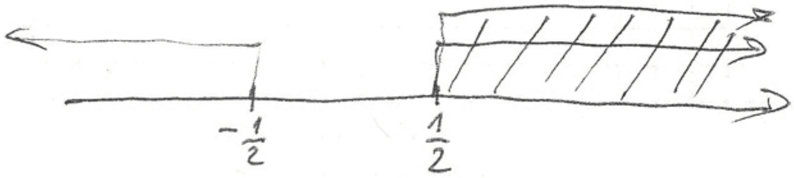
$x \geq \frac{1}{2}$

$2x+1=0 \quad 2x-1=0$

$x=-\frac{1}{2} \quad x=\frac{1}{2}$

	$-\infty$	$-\frac{1}{2}$	$\frac{1}{2}$	$+\infty$
$2x+1$	$-$ 0	$+$	$+$	
$2x-1$	$-$	$-$ 0	$+$	
Rj.	$+$	$-$	$+$	

$x \in (-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, +\infty)$



$\boxed{x \in (\frac{1}{2}, +\infty)}$

KANTONALNO TAKMIČENJE IZ MATEMATIKE
IV RAZRED
Srednjobosanski kanton, školska 2014/15. godina

ZADACI

1.Četvrti član u razvoju binoma $\left(10^{\log\sqrt{x}} + \frac{1}{\log x \sqrt{10}}\right)^7$ je 3500000.
Odrediti vrijednost varjable x.

2.Odrediti kompleksan broj z koji zadovoljava uslove :
$|z+2i| = |z-4i|$ i $|z-4| = 1$

3.Dokazati. Ako je u aritimetičkom nizu suma prvih n članova jednaka n^2p, suma prvih k članova k^2p, Tada je suma prvih p članova jednaka p^3.

4. Izračunati prvi izvod funkcije .
$Y = \ln\left(\sin x + \sqrt{1+\sin^2 x}\right)$

5.Dokazati da je za svako $n \in N$, izraz $A = 5^{2n+3} + 3^{n+3} \cdot 2^n$ djeljiv sa 19.

Vrijeme predviđeno za izradu zadataka je 120 minuta.
Dozvoljena je upotreba samo pribora za crtanje i pisanje.

IV RAZRED

1. $\left(10^{\log\sqrt{x}} + \frac{1}{\sqrt[\log x]{10}}\right)^7$

$T_4 = 35 \cdot 10^5$

$\binom{7}{3} \cdot 10^{4\log\sqrt{x}} \cdot \frac{1}{10^{\frac{3}{\log x}}} = 35 \cdot 10^5$

$10^{4\log\sqrt{x}} \cdot 10^{-\frac{3}{\log x}} = 10^5$

$10^{4\log\sqrt{x} - \frac{3}{\log x}} = 10^5$

$2\log x - \frac{3}{\log x} = 5 \qquad x > 0 \quad \log x = t$

$2t^2 - 5t - 3 = 0$

$t_1 = 3 \qquad t_2 = -\frac{1}{2}$

$x_1 = 10^3 \qquad x_2 = \frac{1}{\sqrt{10}}$

2. $|z+2i| = |z-4i| \quad z = x+yi$

$|z-4| = 1$

$|x+yi+2i| = |x+yi-4i|$

$\sqrt{x^2+(y+2)^2} = \sqrt{x^2+(y-4)^2} \; /^2$

$x^2+y^2+4y+4 = x^2+y^2-8y+16$

$12y = 12$

$y = 1 \quad \Rightarrow$

$|z-4| = 1$

$|x+yi-4| = 1$

$\sqrt{(x-4)^2+y^2} = 1 \; /^2$

$x^2-8x+16+y^2 = 1$

$x^2-8x+16 = 0$

$(x-4)^2 = 0$

$x = 4$

$z = 4+i$

3. $\left.\begin{array}{l} S(n)=n^2p \\ S(k)=k^2p \end{array}\right\} \Rightarrow S(p)=p^3$

$\frac{n}{2}\left(2a_1+(n-1)d\right)=n^2p \,\Big|\cdot\frac{2}{n} \Rightarrow$

$\frac{k}{2}\left(2a_1+(k-1)d\right)=k^2p \,\Big|\cdot\frac{2}{k}$

$\left.\begin{array}{l} 2a_1+(n-1)d=2np \\ 2a_1+(k-1)d=2kp \end{array}\right\} - \Rightarrow$

$(n-\cancel{1}-k+\cancel{1})d=2p(n-k)$

$(n-k)d=2p(n-k)$

$\boxed{d=2p}$

$2a_1+(n-1)\cdot 2p=2np \,|:2$

$a_1+(n-1)p=np$

$a_1+np-p=np$

$\boxed{a_1=p}$

$S(p)=\frac{p}{2}\left(2a_1+(p-1)\cdot d\right)$

$S(p)=\frac{p}{2}\left(2p+(p-1)\cdot 2p\right)$

$S(p)=\frac{p}{2}\left(\cancel{2p}+2p^2-\cancel{2p}\right)$

$S(p)=\frac{p}{2}\cdot 2p^2$

$\boxed{S(p)=p^3}$

4. zadatak

$$y = \ln\left(\sin x + \sqrt{1+\sin^2 x}\right)$$

$$y' = \frac{\left(\sin x + \sqrt{1+\sin^2 x}\right)'}{\sin x + \sqrt{1+\sin^2 x}}$$

$$y' = \frac{\cos x + \frac{\left(1+\sin^2 x\right)'}{2\sqrt{1+\sin^2 x}}}{\sin x + \sqrt{1+\sin^2 x}}$$

$$y' = \frac{\frac{2\cos x\sqrt{1+\sin^2 x} + 2\sin x\cos x}{2\sqrt{1+\sin^2 x}}}{\sin x + \sqrt{1+\sin^2 x}}$$

$$y' = \frac{\frac{2\cos x\left(\sqrt{1+\sin^2 x} + \sin x\right)}{2\sqrt{1+\sin^2 x}}}{\sin x + \sqrt{1+\sin^2 x}}$$

$$y' = \frac{\cos x}{\sqrt{1+\sin^2 x}}$$

5. $19 \mid A$

$$19 \mid 5^{2n+3} + 3^{n+3} \cdot 2^n$$

$$n=1 \Rightarrow 5^5 + 3^4 \cdot 2 = 625 \cdot 5 + 81 \cdot 2 =$$
$$= 3125 + 162 = 3287$$
$$= 173 \cdot 19$$

$n = k$

$$19 \mid \underline{5^{2k+3} + 3^{k+3} \cdot 2^k = 19A}$$

$n = k+1$

$$5^{2(k+1)+3} + 3^{k+1+3} \cdot 2^{k+1} =$$

$$= 5^{2k+3+2} + 3^{k+3+1} \cdot 2^{k+1} =$$

$$= 25 \cdot 5^{2k+3} + 3 \cdot 3^{k+3} \cdot 2 \cdot 2^k =$$

$$= 19 \cdot 5^{2k+3} + 6 \cdot 5^{2k+3} + 6 \cdot 3^{k+3} \cdot 2^k =$$

$$= 19 \cdot 5^{2k+3} + 6\left(5^{2k+3} + 3^{k+3} \cdot 2^k\right)$$

$$= 19 \cdot 5^{2k+3} + 6 \cdot 19A =$$

$$= 19 \cdot \left(5^{2k+3} + 6A\right)$$